Name:	Department: Comp Eng	GRADE
Student No:	Calculus 2 Midterm II	
Signature:	Date: 16/0[redacted]/2017	

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1. Let $f(x,y) = x^2y + e^{xy}$.

   (a) A unit vector in the direction which f increases most rapidly at $(x,y) = (1,0)$ is $\boxed{j}$

   (b) An equation for the tangent plane of f at $(x,y) = (1,0)$ is $\boxed{z-1=2y.}$

a) $\nabla f = (2xy + ye^{xy})i + (x^2 + xe^{xy})j$

$\nabla f(1,0) = 2j \qquad u = \dfrac{\nabla f}{\|\nabla f\|} = j$

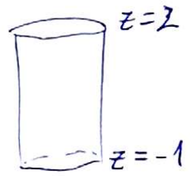
b) $x_0 = 1,\ y_0 = 0,\ z_0 = f(x_0,y_0) = 1.$

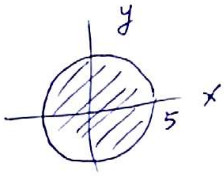
$z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

$z - 1 = 2y$

2. Let D be the region inside the cylinder $x^2+y^2 = 25$ bounded by the planes $z=-1$ and $z=2$.

   $\iiint_D (x^2+y^2)\,dV = \boxed{\frac{3\pi}{2} 5^4}$





$0 \le \theta \le 2\pi$

$0 \le r \le 5$

$-1 \le z \le 2.$

$$\iiint_D (x^2+y^2)\,dV = \int_0^{2\pi}\int_0^5\int_{-1}^{2} r^2\,dz\,r\,dr\,d\theta = 2\pi\cdot(2-(-1))\int_0^5 r^3\,dr$$

$$= 6\pi \left.\frac{r^4}{4}\right|_0^5 = 6\pi \cdot \frac{5^4}{4}.$$

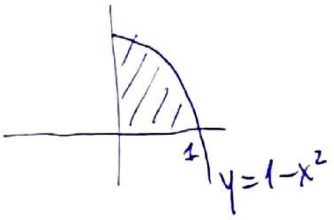
3. $\lim_{(x,y)\to(0,0)} \dfrac{x^2y}{x^4+y^2} = \boxed{\text{DNE}}$ (Write DNE if the value does not exist)

$y = kx^2 \qquad \lim_{\substack{(x,y)\to(0,0)\\ y=kx^2}} \dfrac{x^2y}{x^4+k^2x^4} = \lim_{x\to 0} \dfrac{kx^4}{x^4(1+k^2)} = \dfrac{k}{1+k^2}.$

Since the limit depends on k, $\lim_{(x,y)\to(0,0)} \dfrac{x^2y}{x^4+y^2}$ Does not exist.

4. Let D be the region in the first quadrant bounded by x-axis, y-axis and the curve $y = 1 - x^2$.

$\iint_D x \cos y \, dA =$ $\boxed{\frac{1-\cos 1}{2}}$



$0 \le x \le 1$

$0 \le y \le 1 - x^2$

$$\iint_D x \cos y \, dA = \int_0^1 \int_0^{1-x^2} x \cos y \, dy\, dx = \int_0^1 x \sin(1-x^2)\, dx$$

$$= \int_1^0 \sin u \left(\frac{-du}{2}\right) = \frac{\cos u}{2}\Big|_1^0$$

$1 - x^2 = u \qquad -2x\,dx = du$

$$= \frac{1-\cos 1}{2}$$

5. Let $f(x, y, z) = x + y - z$. On the sphere $x^2 + y^2 + z^2 = 1$, the maximum value of f is $\boxed{\sqrt{3}}$

and the minimum value of f is $\boxed{-\sqrt{3}}$

$g(x,y,z) = x^2 + y^2 + z^2 - 1.$

$\nabla f = \lambda \nabla g \Rightarrow \quad 1 = \lambda 2x, \quad 1 = \lambda 2y, \quad -1 = \lambda 2z$

$x = y = -z = \frac{1}{2\lambda} \Rightarrow x^2 + y^2 + z^2 = 1 \Rightarrow \frac{3}{4\lambda^2} = 1 \Rightarrow \lambda = \mp \frac{\sqrt{3}}{2}.$

$\lambda = -\frac{\sqrt{3}}{2} \Rightarrow \quad x = y = -z = \frac{-1}{\sqrt{3}} \qquad f\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) = \frac{-3}{\sqrt{3}} = -\sqrt{3}$ min.

$\lambda = \frac{\sqrt{3}}{2} \Rightarrow \quad x = y = -z = \frac{1}{\sqrt{3}} \qquad f\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right) = +\sqrt{3}$ max.

6. Find the local maxima and minima and saddle points (if they exist) of the function $f(x, y) = x^2 - xy + y^2 + 2x + 2y - 4$.

$f_x = 2x - y + 2 = 0 \quad \Rightarrow y = 2x + 2$

$f_y = -x + 2y + 2 = 0 \quad \Rightarrow -x + 4x + 4 + 2 = 0 \Rightarrow 3x + 6 = 0 \Rightarrow x = -2$

$\Rightarrow y = -2.$

$(-2, -2)$ is the only critical point.

$f_{xx} = 2, \; f_{xy} = -1, \; f_{yy} = 2.$

$\Delta = f_{xx} f_{yy} - f_{xy}^2 = 4 - 1 = 3 > 0$

$\Delta > 0, \; f_{xx} > 0 \quad \Rightarrow f(-2,-2)$ is min. $\quad f(-2,-2) = 4 - 4 + 4 - 4 - 4 - 4$

$= -8.$